76. (a) We take the x axis along the tunnel, with $x = 0$ at the middle. At any instant during the train's motion, it is a distance r from the center of Earth, and we can think of this as a vector $\vec{r}$ pointing from the train to the Earth's center. We neglect any effects associated with the spinning of Earth (which has mass M and radius R). Based on the theory of Ch. 14, we know that the magnitude of gravitational force on the train of mass m_o at any instant is

$$|F_g| = \frac{Gm_oM\left(r^3/R^3\right)}{r^2} = \frac{Gm_oMr}{R^3} \ .$$

It is only the horizontal component of this force which leads to acceleration/deceleration of the train, so a $\cos\theta$ factor (with θ giving the angle of $\vec{r}$ measured from the x axis) must be included, and we can relate $\cos\theta = x/r$ and obtain

$$m_oa = F_x = -\frac{Gm_oMr}{R^3}\frac{x}{r}$$

where the minus sign is necessary because the force pulls towards the $x = 0$ position, so when the train is, say, at a large negative value of x the force is in the positive x direction (towards the origin of the x axis). The above expression simplifies to exactly the form (Eq. 16-8) required for simple harmonic motion:

$$a = -\omega^2 x \quad \text{where} \quad \omega = \sqrt{\frac{GM}{R^3}} \ .$$

Since a full cycle of the motion would return the train to its starting point, then a half cycle is required to travel from the departure city to the destination city. Therefore, $t_{\text{travel}} = \frac{1}{2}T$.

(b) Since $T = 2\pi/\omega$, we obtain

$$t_{\text{travel}} = \pi\sqrt{\frac{R^3}{GM}} = \pi\sqrt{\frac{\left(6.37\times 10^6\right)^3}{\left(6.67\times 10^{-11}\right)\left(5.98\times 10^{24}\right)}}$$

which yields 2530 s or 42 min.